Testimony About Issues-With-Core Math Standards

R. James Milgram
Professor of Mathematics
Stanford University

Introduction. Controversy surrounds the new Common Core national standards that were created to transform K-12 education in English Language Arts and Math. I was a member of the CCSSO/NGA Validation Committee charged with overseeing the development of these standards. The committee agreed to simply work with the final version and not use its authority to require changes. However, The math standards have areas that are very disturbing,

- For example, they are claimed to be research based, but the main reason I could not sign off on them was that there were too many areas where the writing team could not show me suitable research that justified their handling of key topics - particularly when they differed from standard approaches, in particular the approaches taken in the high achieving countries.
- In too many ways Common Core amounts to a massive experiment with our children – an experiment we are glad Indiana is reconsidering.

Standards are supposed to describe the way in which we bring our children to the level of knowledge that they will need in their lives. Core Standards do not do a good job of this, but as bad as the mathematics Core Standards are, they are significantly better than 90% of the state standards that they replace. Moreover a number of parts are very good indeed.

However, the truth is that core math standards and even the old Indiana math standards – which were better than Core Standards– are not good enough to help make us competitive with the high achieving countries, and we are ever nearer the point where this failure is going to cost us catastrophically.

- Indeed, the recent mortgage crises that almost brought us to our knees -- which can be directly tied to the fact that we no longer teach compound interest in K-12 – is only a precursor of what is likely to be coming. (Actually, we managed to get compound interest as a seventh grade standard in California's 1997 standards, but it turned out that middle school teachers could not generally handle it.)
- As this example dramatically shows, the biggest part of the problem is precisely with our mathematical outcomes.

Most of the high achieving countries in mathematics don't have detailed standards: they just mention the topics to be covered, and the hearts of their results are determined by the extraordinary books and curricula that they use to fill out the sketches in their standards. Why not just adopt their books? Unfortunately, here is where we run into real problems.

- For example, by far the most accessible curricula from any of the high achieving countries, – that of Singapore, which is even written in English – is typically not successful when it is piloted here.
- The main reason is that, though the books appear to be elementary, this is just a reflection of the care taken in their writing. They are not remotely elementary.
- What too many of our teachers find is that all too soon, they are no longer able to solve the student exercises, and their teaching falls apart.
- This is the case even in states with standards that compare well with the Singapore expectations.

As scary as these issues are, perhaps even worse are the extremely expensive and highly non-standard tests that are currently being designed to go along with Core Standards. Indiana has essentially withdrawn from the exams, and last week alone Georgia and Oklahoma withdrew. Moreover, Florida seems ready to as well.

Basic Issues. International expectations in a number of the high achieving countries are that a calculus course is required to graduate from high school, and over 90% of their citizens have high school degrees. In the high achieving countries where calculus is not required, the data seems to indicate that at least 50% of high school graduates have had calculus. In the U.S. This number is arounc 20%, and as we will see, is only likely to decrease under Common Core. *Indeed, the College Board has begun to explore possibly revamping the AP calculus exams or replacing them by lower level exams. I do not know how the universities will react to this, but it is likely that if this happens, many will stop giving calculus credit for them.*

Supporters insist that Common Core is the only way to address the problem of constantly declining U.S. Student outcomes and trying to match up with our international competitors.
In contrast to standards like the previous Indiana, Massachusetts and California standards, Core Standards only covers material from Algebra I, Geometry and Algebra II. It does not cover more advanced material in trigonometry or linear algebra, let alone pre-calculus, or calculus even though more and more U.S students are beginning to arrive at university with AP credit in Calculus, and this has affected university expectations for entering students.

As a result the Common Core math standards fall far short of international expectations and of what students need for more advanced work, or to enter the workforce in any mathematically intensive area.
> One of the main authors of the Core Math Standards, Jason Zimba, testified to the Massachusetts State Board of Education in 2010 that Common Core is only designed to prepare students for an entry level job or a non-selective community college, not a four year university.[1]

For what it's worth, in conversations with Zimba and Bill McCullum, when I was acting in my role as a member of the validation committee to oversee what they were doing, they described what they were trying to do in very similar words to Zimba's remarks in Massachusetts. They did not specifically say "community college," but the material they were including in core standards was not sufficient to prepare students for calculus. It was clear that the remedial courses they were talking about were, at best, at the level of "Intermediate Algebra," which is not even offered as a credit bearing course at the vast majority of 4 year state universities.

The (+) Standards in CCSSI. The base model for Core Standards in mathematics was Achieve's ADP standards. There are two types of ADP math standards, those without an asterisk and those with an asterisk. Those with asterisks were the standards judged essential to successfully enter a regular 4 year university intending to major in any area that requires college level mathematics or to enter the workforce with a math intensive job. Those without asterisks were for students directly entering the workforce after high school or those with no intent to major in an area that does require more serious mathematics. The ADP asterisk-standards that are present in Core Standards are represented with a (+).

Here is what Core Standards says about the (+) standards:
> "The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by

---

1   http://www.doe.mass.edu/boe/minutes/10/0323reg.doc In the section titled "Mathematics." Mr. Zimba said the idea of a third pathway is a good one, and could show calculus as one of the pathways. Mr. Zimba said the concept of college readiness is minimal and focuses on non-selective colleges…."

(+)."
So far, so good. But then Core Standards goes on to say:
> "All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards without a (+) symbol may also appear in courses intended for all students."

The problem is that the CCSSI (+) standards are comprised entirely of topics that are normally in regular high school Algebra I, Algebra II, or Geometry courses. And the above makes it clear that the expectation is that (all) students are not expected to even be exposed to this material, but it is implicitly recommended that the high schools should track students: taking the "more able ones" and putting them in more advanced or highly enriched courses. However, even when the (+) topics are put in, there is virtually none of the material that would be in a regular pre-calculus course or more than the beginning weeks of an actual course in linear algebra (*aside from the fact that the (+) standards in this area contain many errors, since the authors seem to confuse vectors with vector fields*). Similarly, there is nowhere near the material that would be in a standard trigonometry course. Nor is there any material that would be in a course on solid geometry, an old topic that is recently becoming more important.

It is also worth pointing out what PARCC says about the meaning of succeeding in the PARCC mathematics exam:
> Students who earn a College- and Career-Ready Determination in mathematics will have demonstrated the academic knowledge, skills and practices necessary to enter directly into and succeed in entry-level, credit-bearing courses in College Algebra, Introductory College Statistics, and technical courses requiring an equivalent level of mathematics.[2] See

Of course, "college algebra" is the beginning math course at community colleges. The beginning level course at most state universities is pre-calculus, but at more and more universities in this country even the ordinary calculus course is now regarded as remedial.

The major mathematical issues with Core Standards. The three most severe problem areas are
1. the beginning handling of whole numbers in particular understanding place value notation and then adding, subtracting, multiplying, and dividing;
2. the handling of geometry, ratio, rates, proportions and percents in middle school and high school;
3. the very low level expectations for high school graduation that barely prepare students for attending a community college, let alone a 4-year university.

Unfortunately, these are the three most crucial areas, aside from fractions, where our math outcomes have to improve. We have already discussed the third issue. So in the remainder of this section we focus on the first and second.

Whole numbers. Core Standard's approach to whole numbers is just the continuation of the approach pioneered in California in the early 1990's. *It had such bad outcomes in California that it spawned the Math Wars.* A cornerstone of the approach is that students begin by constructing their own algorithms[3]. But the use of student-constructed algorithms is at odds with the practices of high-achieving countries, and the research that supports student constructed algorithms appears highly suspect.

CCSSI is prescriptive about teaching methods in the math standards, particularly in the early grades and in the discussion of fractions. But most of the required pedagogy does not conform at all with the

---

2   http://www.parcconline.org/sites/parcc/files/PARCCCCRDPolicyandPLDs_FINAL_0.pdf

3   On page 13 of the Mathematics Core Standards we have "In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; There are many such statements." They are euphemisms for "student constructed algorithms."

practices in the high achieving countries.
1. Besides the "fuzzy math" ideas implicit in student constructed algorithms, the handling of fractions prescribes the use of what CCSSI calls "visual fraction models."
2. However, there is research by professional mathematicians as well as math educators in the USSR indicating that it is precisely the use of these models that leads to our horrible outcomes with fractions.

Here is an example of why this research required professional mathematicians.
- Researchers in the U.S. math education community claimed, based on their study of actual Japanese lessons, that the Japanese used student discovery methods extensively.
- When actual mathematicians looked at the particular lesson that the U.S. math educators had pointed at to justify their conclusions, the mathematicians found that at the beginning of the lesson – where the math educators thought the teacher was just fooling around – he had actually laid out the material underlying the days lesson.
- At the same time he also laid out the methods needed to solve the problem he presented a bit later in the lecture.[4]

This indicates another issue with the education research: the lack of subject matter knowledge on the part of the mathematics education researchers in the U.S.

Geometry. The way Common Core presents geometry is not research based, and the only country that tried this approach on a large scale[5] rapidly abandoned it. The problem is that even though the outlined approach to geometry is rigorous, it depends on highly specialized topics that even math majors at a four year university would not see until their second or third years. Again, there is no research that supports the Core Standards approach.

Tied in with the problems in geometry, there are also serious problems with the way Common Core handles percents, ratios, rates, and proportions – the critical topics that are essential if students are to learn more advanced topics such as trigonometry, statistics, and even calculus.[6]

Algebra. In addition to the deficiencies above, Common Core only includes most (but not all) of the standard algebra I expectations, together with only some parts of standard geometry and algebra II courses. There is no content beyond this, and this is exactly where the fact becomes apparent that it is designed to only prepare students for entry level jobs or entering a 2 year college after high school.

This is expanded on in detail by the well known mathematician, Johnathan Goodman, of the Courant Institute at NYU in his report on the comparison of Core Standards with the standards of the high achieving countries.[7]

Further mathematical details. The classic method of, for example, adding two-digit numbers is to add the digits in the "ones" column, carry the tens in the sum to the "tens" column, then add the "tens" digits, and so on. This "standard algorithm" works first time, every time. But instead of preparing for and teaching this method by first carefully studying and understanding the meaning of our place value notation as they do in the high achieving countries,

---

4 Alan Siegel, Proceedings of International Congress of Mathematicians, 2006
5 The old USSR
6   Among other things, it is usual in the high achieving countries to begin the study of these topics in third or fourth grade, but in Core Standard it is only covered in two sections, one in sixth grade and one in seventh. After this, ratio is never mentioned again except once in the first Algebra standard, A-SSE.
7 Johnathan Goodman, *A comparison of proposed US Common Core math standard to standards of selected Asian countries.* http://www.educationnews.org/ed_reports/94979.html#sthash.MQlFLz64.dpuf

1. Common Core has a relatively superficial discussion of place value and place value notation that is consistent with the usual U.S. practice of treating numbers written in place value notation as reading words.
2. In the U.S., students learn that the important thing is to be able to read 311 as "three hundred eleven" rather than emphasizing that it means "the sum of 3 copies of 100, one copy of 10 and a 1" (the keyword being sum) as is carefully focused on in the high-achieving countries.
3. In fact, item 2 is embedded in their language and, as a result, is automatic in the high achieving countries. However, it has to be given serious attention in each of the first 3 or four grades in the U.S. if our outcomes are to improve.

But Common Core ignores the fact that students, typically, have not internalized the place value notation representation of whole numbers[8] and creates a three-step process starting with student constructed algorithms. This process is guided by the absurd belief that all this content – which took over 200 years to fully develop – is somehow innate.

Finally, *but years later than is the case in the high achieving countries*, it is simply required that students are able to use the standard algorithms without having been taught the key parts of the material that explains why and how they work. I have to admit that even mentioning the standard algorithms is very unusual in state standards (though they were certainly present in the previous California standards)[9]. This is a strength of Core Standards. However, there is no support in the remainder of Core Standards for student understanding. The total discussion of the standard algorithms is contained in the following three standards and is woefully incomplete.

1. In fourth grade: "Fluently add and subtract multi-digit whole numbers using the standard algorithm."
2. "In fifth grade: Fluently multiply multi-digit whole numbers using the standard algorithm."
3. Finally in sixth grade, when students in other countries are already beginning to study algebra, we have: "Fluently divide multi-digit numbers using the standard algorithm.

Let me emphasize again that each of these standards appears at least two years later than in the high achieving countries.

Sandy Stotsky strongly suggests, and I strongly support the suggestion, that for any state thinking of implementing CCSSI in mathematics

> the legislature and/or governor commission a group of engineering, math, and science faculty from the state's own universities and colleges to work together with a group of high school math teachers on secondary standards that lead to the kind of coursework they want kids to be able to take (and pass) before applying for admission to their colleges/universities. The higher ed people on this commission should be those teaching college freshmen math, science, reading, or English (not administrators or education faculty).

But of course even this minimalist suggestion runs counter to CCSSI's requirements that only at most

---

8   Recent research using internal data from a major testing company, indicates that over 50% of second and third graders in this country do not understand the place value notation.
9   California Mathematics Content Standards adopted Dec,1997
Grade Four
3.0 Students solve problems involving addition, subtraction, multiplication, and division of whole numbers and understand the relationships among the operations:
3.1 Demonstrate an understanding of, and the ability to use, standard algorithms for the addition and subtraction of multidigit numbers
3.2 Demonstrate an understanding of, and the ability to use, standard algorithms for multiplying a multidigit number by a two-digit number and for dividing a multidigit number by a one-digit number; use relationships between them to simplify computations and to check results.
Grade Five
2.2 Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors.

15% of a state's standards are allowed to not be in the common core.

Conclusions. Let me say this again: *the objective of the Common Core Math Standards is to present the minimal amount of material that high-school graduates need to be able to enter the work force in an entry-level job, or to enroll in a community college with a reasonable expectation of avoiding a remedial math course.* There is no preparation for anything more, such as entering a university (not a community college) with a reasonable expectation of being able to skip the entry-level courses. (Virtually no university student who has to take an entry-level math course ever gets a degree in a technical area such as the hard sciences, engineering, economics, statistics, or mathematics.)

The authors seem to believe that the "best" students can be tracked into more advanced classes that will teach more standard material. But the reality is that even though the authors seem to believe that Core Standards are a floor, in practice they will soon become a ceiling. This is especially true since there will be standardized tests involved which tend to control the actual curriculum and we can already see, from the released sample items, that the intention is to make these tests very low level.[10]

I cannot emphasize enough that Common Core is using our children for a huge and risky experiment, one that consistently failed when tried by individual states such as California in the early 1990's and even countries such as the old USSR in the 1970's.

---

10 See the next section

The Common Core tests Currently being Prepared.

Why have states like Georgia and Oklahoma opted out of the common core test consortia? First the tests are going to be extremely expensive, and second they are so experimental that there is absolutely no data showing that their results will give us any real information on what mathematics students understand and can use.

Additionally, at least in math, the questions tend to be so poorly designed that it is not at all clear what they are even testing.

Below are four examples taken from the SBAC sample exams because the PARCC consortium has not yet released sample items and exams. All the examples require students to extensively use their mouse which leaves little room to test high level mathematical material. As a result the questions only test the most routine mathematics at each grade level. But the third example is more problematic. It is actually incorrect precisely because the authors do not understand their computer interface.
It is worth studying these examples as they provide a clear indication of both the intended level of the test questions and the sloppy approach to mathematics they convey. I repeat a question I ask after the third example: *If students are comfortable ignoring errors like those in example three, how would you like to have them designing things like a manned lunar probe or even a simple high rise building?*

Here is the first example

SMARTER
Balanced Assessment Consortium

Grade 05 Mathematics Sample TE Item

Using estimation, classify each product below as less than $\frac{5}{8}$ or greater than $\frac{5}{8}$ by moving the product to the correct box.

|				
$\frac{5}{8} \times \frac{1}{4}$	$\frac{5}{8} \times \frac{13}{6}$	$\frac{5}{8} \times 1\frac{1}{16}$	$\frac{5}{8} \times \frac{7}{8}$	$\frac{5}{8} \times 3$

Less Than $\frac{5}{8}$	Greater Than $\frac{5}{8}$

Analysis: There are two things being tested here, students ability to "mouse" and one relatively routine fact about numbers -- when you multiply a positive number, a, by a number, b, that is greater than 1, then ba > a, and if b is less than 1, then ba < a. Mousing is something that is dependent on the resources available state-wide, and cannot be assumed. The basic math fact being tested is relatively low level, typically third or at most fourth grade material in the high achieving countries.

Below is a sixth grade item. Once more, there are just two things going on. The first is students ability to "mouse," and the second is that the students are able to find coordinates in two of the four quadrants of the coordinate plane. However, the statement of the problem -- artificially embedding the tested math skill in a "story problem" -- fails dramatically. The biggest issue is that a grocery store is not really a point. It takes up space, and so we could, perhaps specify that coordinates (-2, -4) give the position of the upper left corner of the store, and we want to specify the coordinates of the upper left

22. The map of a town is placed on a coordinate grid with each whole number distance north (N), south (S), east (E), or west (W) representing 1 block.

A grocery store has the coordinates (-2, -4). The owners of the grocery store plan to build an additional grocery store at a location that is 5 blocks to the east and 3 blocks to the north of the original store. Plot the location of the additional grocery store on the coordinate grid.
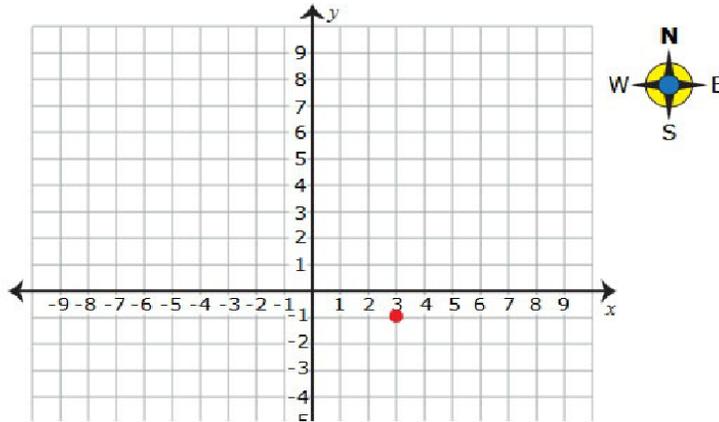
corner of the new store .... But as it is stated, students who have seen similar problems incorrectly developed in class know that the hidden assumption here is that we can represent a store by a single point, whereas a student who as been correctly taught to make minimal assumptions and to clearly be able to articulate the assumptions that are being made would have serious difficulties with this problem.

SMARTER
Balanced Assessment Consortium

Classify each pair of shapes below as a pair of congruent shapes, a pair of similar shapes, or a pair of shapes that is neither congruent nor similar by moving each pair of shapes to the correct box.
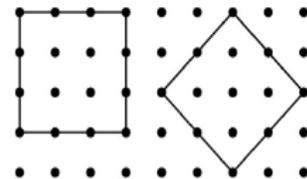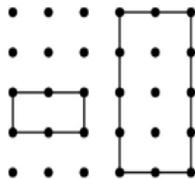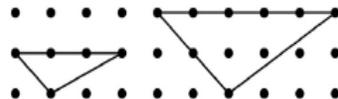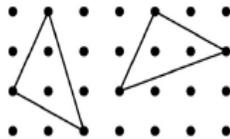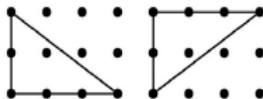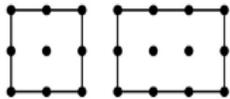
Pairs of Congruent Shapes	Pairs of Similar Shapes	Pairs of Shapes That Are Neither Congruent Nor Similar

Here is an eighth grade item that , at first glance, seems little more than a test of vocabulary and skill with a computer mouse. But on second glance we see the hidden assumptions. If, as was the case with the problem above, the student knows the hidden assumptions underlying the statement of the problem, namely that the dots are exactly the points on an integer coordinate grid .where the vertical distance is $|y_1 - y_2|$ and the horizontal distance is measured by $|x_1 - x_2|$. However, it turns out that in the actual pictures the vertical distance between grid points, $(x, y+1)$ and (x,y), is about 1.18 times that of the horizontal distance, $(x+1,y)$ and (x,y). So, once more, a properly taught, careful student would be at a serious disadvantage. Moreover, this actually matters. *If students are comfortable ignoring errors in the range of 20%, how would you like to have them designing things like a manned lunar probe?*

Finally, here is a second eighth grade sample item, which appears to be almost as trivial as the one above, but at least this one appears to be correct, even if the major skill being demonstrated is again "mousing."

Mathematics Sample TE Item

Use the numbers in the box to make the equations below true. The numbers cannot be used more than once. Click on a number and then drag it to the appropriate box.

| 4 | 8 | 10 | 64 | 100 | 1,000 |

$\sqrt{} = $ $\sqrt[3]{} = $